Resonant Circuits and Soft Switching
(LLC Resonant Converter and Resonant Inverter)

Outline:
This document discusses the principles of resonant circuits and soft switching and describes application examples of LLC resonant converters and resonant inverters (an inductive-heating circuit and a discharge tube drive).
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1. Resonant circuits

Resonant circuits are used in power supply circuits for one or more of the following purposes: high frequency output of voltage or current, switching loss reduction, and switching noise reduction.

This application note discusses resonant circuits primarily designed for high-frequency AC generation. In addition, it discusses soft switching that is mainly used to reduce switching losses.

1.1. Resonant circuits

A resonant circuit is a type of electric circuit that produces vibrations in response to externally applied energy. Resonant vibrations are used by various electric circuits.

Basically, a resonant circuit comprises a coil (L) and a capacitor (C). There are two types of resonant circuits, depending on their topology: parallel and series resonant circuits. At the resonant frequency, parallel resonant circuits have effectively infinite impedance whereas series resonant circuits exhibit zero impedance.

The energy loss of a resonant circuit is represented by a parameter called a quality factor (Q factor). A higher Q factor indicates a lower rate of energy loss and therefore a more preferable resonator. Care should be exercised, however, because resonant circuits with a high Q factor might not go into resonance or require a long period of time to become stable depending on their applications if their frequency deviates from the resonant frequency, even slightly.

In the case of resonant inverters, the Q factor of a resonant circuit varies dramatically with the load impedance. The Q factor of such a resonant inverter is expressed as a ratio of the load impedance to the LC impedance and is therefore affected by the load impedance. The optimum topology of the resonant inverter depends on the load impedance. Figure 1.1 shows three typical types of resonant circuits used in resonant inverters.

![Figure 1.1 Resonant circuits](image)

The parallel LC circuit of Figure 1.1(a) is used when a load has large impedance. Assuming that impedance is very large, this LC circuit goes into resonance properly since it can be viewed as equivalent to a circuit consisting only of an inductor (L) and a capacitor (C). Conversely, suppose that load is small, C has little effect. Consequently, this circuit does not go into resonance because it is equivalent to only an inductor connected to GND.

In the series LC resonant circuit of Figure 1.1(b), a smaller impedance value makes it function in a manner close to a resonant circuit comprising only an inductor and a capacitor, and therefore
provides high Q. Conversely, if impedance is large, Q becomes smaller, making the series LC circuit unsuitable for series resonance.

The LCC resonant circuit of Figure 1.1(c), which is a hybrid of parallel and series LC circuits, works as a resonant circuit regardless of the value of impedance.

As described above, the optimum type of resonant circuit must be selected according to the load.

2. Soft switching (for reducing switching losses and switching noise)

Partial resonance, one of the techniques for soft switching, turns on and off power switching devices at zero voltage or current by using a resonance phenomenon. In partial resonance, the switching loss become very small because either voltage or current is zero when they switch on or off.

Switching at zero-voltage crossing is called **zero-voltage switching (ZVS)** whereas switching at zero-current crossing is called **zero-current switching (ZCS)**.

With the typical switching method called **hard switching**, there is a considerable overlap between voltage and current waveforms. In contrast, soft switching helps reduce losses caused by power device switching. In addition, di / dt and dv / dt lamps are reduced, which is effective in reducing switching harmonic components and transient noise.

**Applications of soft switching**

Soft switching is used for various switched-mode power supplies. For switch-mode power supplies, it is important to reduce power loss to increase efficiency. In addition to the turn-off loss of switching devices, soft switching helps reduce transformer leakage inductance loss and diode recovery loss.

It is necessary to consider both the advantages and disadvantages of soft switching to determine whether it is an appropriate technique for your application. While soft switching has the advantages of reduced switching losses and high-frequency noise, it has disadvantages such as an increase in parts count and a need for more complicated control.

Soft switching is used for switched-mode power supplies having a high-frequency transformer such as isolated DC-DC converters (LCC resonant converters).

Since high-frequency transformers have leakage inductance, they cause a reduction in the efficiency of switched-mode power supplies when the energy stored in the leakage inductance is dissipated by the switching of transistors. Soft switching also helps reduce the losses caused by the leakage and excitation inductances of a high-frequency transformer. In addition, soft switching incurs a minimal increase in the parts count and cost because the leakage inductance of a high-frequency transformer can be used as an inductor for resonance.
Soft-switching circuit topology

The following paragraphs describe the basic circuit topology and the operation of partial resonance. Figure 2.1 shows relatively simple half-bridge inverters.

![Figure 2.1 Switching techniques](image)

(a) Hard-switching circuit  
(b) ZVS circuit  
(c) ZCS circuit

The **hard-switching** circuit shown in Figure 2.1(a) uses a typical switching technique.

The **ZVS topology** connects C1 and C2 in parallel with switching devices as shown in Figure 2.1(b). This circuit achieves switching at zero voltage, resonating with load and other inductances.

The **ZCS topology** connects L1 and L2 in series with switching devices as shown in Figure 2.1(c). This circuit achieves switching at zero current, resonating with load and other capacitances.

Because of the reduced di/dt and dv/dt ramps during switching transitions, the ZVS and ZCS techniques help reduce switching losses as well as the influence on the circuit and the noise generated by, switching devices.

**ZVS inverter operation**

**Turn-off operation**

In Figure 2.2, Q1 and Q2 are MOSFETs, DQ1 and DQ2 are MOSFET body diodes, and C1 and C2 are the sums of the parasitic and external capacitances of Q1 and Q2, respectively.

When Q2 turns off, the Q2 voltage begins to rise, causing the current flowing through Q2 to circulate to C2. Consequently, the C2 voltage (vQ) increases gradually. The voltage slope depends on the capacitance of C2. A higher C2 value results in a shallower vQ slope and therefore lower switching loss. Figure 2.3 shows the current and voltage waveforms for different C2 values.
Figure 2.2 ZVS turn-off operation

Figure 2.3 ZVS turn-off waveforms
Turn-on operation

Next, the following paragraphs discuss the ZVS turn-on of switching devices. When Q1 is on and Q2 is off, Q2 turns on as described below immediately after Q1 turns off. Charge still remains in capacitor C2 immediately after Q1 turns off. Q2 would perform hard switching if it were turned on in this state.

To achieve soft switching, it is necessary to remove the charge from C2 to allow diode DQ2 to conduct (vQ=V_F) and then turn on Q2. Figure 2.4 shows this sequence:

(a) Charge is removed from C2.

The energy stored in L is used to remove charge from C2. (When Q1, a high-side switching device, switches from the on to the off state, current flows as shown by the arrow in (a) owing to the energy stored in L.)

(b) DQ2 conducts.

When charge is completely removed from C2, current passes through DQ2.

(c) Q2 turned on.

Q2 is turned on while DQ2 is conducting. Since vQ is nearly 0 V at this time, zero-voltage switching (ZVS) occurs.

(d) On state

When the energy stored in load L is released, the current passing through Q2 reverses its direction, putting Q2 back in the normal “on” state, with current flowing from drain to source. At this time, Q2 is on, and its voltage (vQ) is nearly 0 V. Therefore, Q2 achieves soft switching.

![Basic ZVS circuit](image)

(a) Charge is removed from C2.

(b) DQ2 conducts.

(c) Q2 turns on.

(d) Q2 is on state.

Figure 2.4 ZVS turn-on operation
3. LLC resonant converter

A DC-DC converter (power supply) requires downsizing, high efficiency, high output power and low EMI noise. To create such a power supply, a circuit topology that combines high-frequency switching and high efficiency is necessary. Previously, PWM (Pulse Width Modulation) control was commonly used for converters. However, the increase in switching frequency has caused problems such as increased switching loss due to turn-on and turn-off and generation of high-frequency noise.

An LLC resonant converter, which combines a current-resonant operation and soft switching (partial resonance), provides several desired features such as low parts count, high efficiency, and low noise. The name “LLC” is derived from the leakage inductance ($L_r$) and excitation inductance ($L_m$) of a transformer and a capacitor ($C$) that are used to achieve resonance. (For leakage and excitation inductances, see Supplement 1 and Supplement 2, respectively.) Figure 3.1 shows an LLC resonant converter, which is composed of a half-bridge (i.e., a square-wave generator) consisting of two series MOSFETs ($Q_1$ and $Q_2$), a resonant capacitor ($C_r$), a transformer ($T$), two output rectifier diodes ($D_1$ and $D_2$), and an output capacitor ($C_o$). In Figure 3.1, $N_1$ represents the number of turns in the primary winding whereas $N_2$ and $N_3$, which are equal, represent the numbers of turns in the secondary windings. $L_r$ is the leakage inductance of the primary winding of the transformer. Generally, a transformer with a low coupling coefficient is used in an LLC resonant converter to provide large leakage inductance so that the leakage inductance acts as resonant inductance. (For the coupling coefficient, see Supplement 3.) In some cases, a separate inductor is connected in series with a transformer. $L_m$ represents excitation inductance.

Even in the presence of input voltage variations, an LLC resonant converter makes it possible to reduce the control frequency range through the use of two different resonant frequencies: a fixed resonant frequency ($f_r$) of an inductor-capacitor ($L_r-C_r$) pair and a resonant frequency ($f_m$) derived from the inductor-capacitor pair ($(L_r + L_m)-C_r$) that varies with load $R_o$.

The LLC resonant converter generates a square-wave voltage at a duty cycle close to 50%, which is converted into a nearly sinusoidal current with an LLC resonant circuit. Since the LLC resonant converter suppresses harmonics and allows ZVS soft-switching operation, it is widely used for applications requiring high efficiency and low EMI noise.
Figure 3.1 LLC resonant converter circuit
**Description of the LLC resonant converter**

An LLC resonant converter contains a square-wave generator and a resonant circuit. As shown in Figure 3.1, an LLC resonant converter is basically composed of: a) a square-wave generator circuit, b) a resonant circuit, and c) a rectifier and an output circuit.

**(a) A square-wave generator circuit**

Two MOSFETs (Q1 and Q2) alternately turn on and off at a frequency of f. The square-wave generator generates a square-wave voltage with an amplitude of ±Vin/2 from a DC power source with a voltage of Vin. To prevent a short-circuit between Q1 and Q2, a dead time is inserted between their on and off transitions. ZVS soft switching is performed during this period.

**(b) A resonant circuit**

The leakage inductance (Lr) of the primary winding of the transformer, the excitation inductance (Lm) of the transformer and capacitance (Cr) form a series resonance circuit. The excitation inductance (Lm) also forms a parallel circuit with the load resistance (Ro'), which equivalently represents the circuit on the secondary side of the transformer as resistance on the primary side (Figure 3.2).

An LLC resonant circuit has two different resonant frequencies: a fixed resonant frequency (fr) that is a function of Lr and Cr and a resonant frequency (fm) that is determined by (Lr + Lm) and Cr. (fm varies with Ro’ connected in parallel with Lm.)

![Figure 3.2 Equivalent circuit for LLC resonance](image)

**(c) A rectifier and an output circuit**

The last stage of the LLC resonant converter consists of a full-wave rectification circuit made up of two diodes, followed by an output capacitor (Co). Schottky barrier diodes with low forward voltage and fast reverse recovery time may be used as rectifier diodes, or MOSFETs may be used for synchronous rectification instead of diodes. Since the cathode of each diode is connected to the capacitor (voltage Vo), a square-wave voltage with an amplitude of ±Vo appears across the secondary winding of the transformer. The voltage across the primary winding also has a square waveform with an amplitude of ±N·Vo (where N = N1/N2).
3.1. Overview of the basic primary-side operation of the LLC resonant converter

The following paragraphs describe the basic current resonance operation of the LLC resonant converter. Figure 3.3 shows a schematic and current paths of the primary side. Figure 3.4 shows its basic waveforms (see Figure 3.5 for more details). This circuit is an LC series resonance circuit, which exhibits current resonance.

#1. Q₁ turns on, passing iQ₁.
#2. When Q₁ turns off, current (iQ₂) flows through the body diode of Q₂ in the reverse direction. Q₂ is turned on while current is flowing through the body diode.
#3. When the capacitor current (iCr) changes its direction from positive to negative because of LC resonance, iQ₂ flows through Q₂ in the positive direction.
#4. When Q₂ is turned off while iQ₂ is positive, current (iQ₁) flows through the body diode of Q₁ in the reverse direction. Q₁ is turned on while current is flowing through the body diode.

At Step 2, current is passed through the body diode of Q₂, and then Q₂ is switched on when its voltage has reached almost zero (ZVS). Likewise, Q₁ undergoes zero-voltage switching at Step 4.

Figure 3.3 Current paths in the LLC resonant converter

Figure 3.4 Basic waveforms of the LLC resonant converter
3.2. Detailed description of the LLC resonant converter operation

Figure 3.5 shows the current resonance operation of the LLC resonant converter, which is a detailed view of the waveforms shown in Figure 3.1. This section also details the operation of this circuit.

Figure 3.5 Basic waveforms of the LLC resonant converter
Operating mode 1-2: Q₁ is on, and Q₂ is off (Figure 3.6).

Q₁ is on, and Q₂ is off. The output current from the MOSFET half-bridge (i_r) flows through L_r and C_r and becomes the primary winding current of the transformer (T), which consists of the excitation current (i_m) and the primary winding load current (i_o1). On the secondary side, current flows to C_o via diode D₁. Since the secondary-side output of the LLC circuit is connected to a low-impedance capacitor (C_o), DC output voltage (V_o) is applied across the secondary winding of the transformer (T), causing a voltage of (N₁/N₂) \cdot V_o to appear across the primary winding according to the transformer's turns ratio (N = N₁/N₂).

Of the primary winding current (i_r) in the transformer (T), the primary winding load current (i_o1) has a sine waveform, which is determined by the resonant circuit consisting of L_r and C_r. As the voltage applied to the excitation inductance has a square waveform, the excitation current (i_m) for the transformer (T) increases linearly. Therefore, the load current (i_o1), which is equal to (i_r – i_m), has a waveform close to a sinusoidal wave. The LLC resonant converter transitions to the next mode when the primary winding load current (i_o1) has reached zero.

Operating mode 2: Q₁ is on, and Q₂ is off (Figure 3.7).

Q₁ remains on and Q₂ remains off. Because the primary winding load current (i_o1) becomes zero (upon termination of the resonance between C_r and L_r), only the excitation current (i_m) flows into the transformer. Then, Q₁ turns off, and the LLC resonant converter transitions to the next mode when the flow of current (i_Q₁) is finished.

Operating mode 3: Both Q₁ and Q₂ are off (Figure 3.8).

Now, both Q₁ and Q₂ are off. The excitation current (i_m) keeps flowing into the transformer via the parasitic capacitances (C_Q₁ and C_Q₂) of Q₁ and Q₂, charging C_Q₁ and discharging C_Q₂. This causes the Q₁ voltage (V_DSQ₁) to increase and the Q2 voltage (V_DSQ₂) to decrease. The LLC resonant converter transitions to the next mode when the charging and discharging of C_Q₁ and C_Q₂ finish.
Operating mode 4: Both $Q_1$ and $Q_2$ are off (Figure 3.9).

The excitation current ($i_m$) continues flowing even after the charging and discharging of $C_{Q1}$ and $C_{Q2}$ finish. The excitation current flows through the body diode of $Q_2$. The LLC resonant converter transitions to the next mode when $Q_2$ turns on.

![Figure 3.8 Operating mode 3 of the LLC resonant converter](image)

![Figure 3.9 Operating mode 4 of the LLC resonant converter](image)

Operating mode 5-1: $Q_1$ is off, and $Q_2$ is on (Figure 3.10).

$Q_1$ remains off, and $Q_2$ turns on. At this time, $C_r$ holds charge at high voltage. $Q_2$ turns on while $D_{Q2}$ is freewheeling under the zero-voltage-switching (ZVS) condition.

When $Q_2$ turns on, $C_r$ acts as a power source because of the stored charge, causing the primary winding load current ($i_{O1}$) to flow through the primary winding of the transformer ($T$) in the reverse direction. As a result, a voltage of $-V_0$ appears across the secondary winding in reverse polarity through diode $D_2$, causing a negative voltage of $-(N_1/N_3)\cdot V_0$ to be applied across the excitation inductance ($L_m$) on the primary side of the transformer ($T$). This voltage causes the excitation current ($i_m$) to decay.

Operating mode 5-2: $Q_1$ is off, and $Q_2$ is on (Figure 3.11).

The excitation current ($i_m$) crosses the zero point and increases in the negative direction. After a while, the resonance between $C_r$ and $L_r$ terminates. The LLC resonant converter transitions to the next mode when the primary winding load current ($i_{O1}$) becomes zero.
Operating mode 6: Q1 is off, and Q2 is on (Figure 3.12).
Q1 remains off, and Q2 remains on. The primary winding load current (iO1) is zero; so only the excitation current (i_m) continues flowing into the transformer (T) via L_r and C_r, with the resonant capacitor (C_r) acting as a power source. The LLC resonant converter transitions to the next mode when Q2 turns off.

Operating mode 7: Both Q1 and Q2 are off (Figure 3.13).
Since Q2 has turned off, both Q1 and Q2 are off at this point. However, the excitation current (i_m) continues flowing into the transformer (T) via the parasitic capacitances (C_Q1 and C_Q2) of Q1 and Q2, causing C_Q2 to be charged and C_Q1 to be discharged. Consequently, the Q2 voltage (V_DSQ2) increases, and the Q1 voltage (V_DSQ1) decreases. The LLC resonant converter transitions to the next mode when the charging and discharging of C_Q1 and C_Q2 finish.
Operating mode 8: Both Q₁ and Q₂ are off (Figure 3.14).

The excitation current (iₘ) continues flowing even after the charging and discharging of C_Q₁ and C_Q₂ finish. The excitation current flows through the body diode of Q₁. The LLC resonant converter transitions to the next mode when Q₁ turns on.

Operation 1-1: Q₁ is on, and Q₂ is off (Figure 3.15).

Q₂ remains off, and Q₁ turns on. At this time, Cᵣ holds charge at high voltage. Q₁ turns on while D_Q₁ is freewheeling under the zero-voltage-switching (ZVS) condition.

When Q₁ turns on, Cᵣ acts as a power source, causing the primary winding load current (i₀₁) to flow through the primary winding of the transformer (T). As a result, a voltage of V₀ appears across the secondary winding in reverse polarity via diode D₁, causing a voltage of (N₁/N₂)·V₀ to be applied across the excitation inductance (Lₘ) on the primary side of the transformer (T). This voltage causes the excitation current (iₘ) to decay.

3.3. Controlling the output voltage of the LLC resonant converter

To control the output voltage, an LLC resonant converter varies the MOSFET switching frequency (f) by using the gain-frequency characteristics of an LLC resonant circuit, which exhibits two different resonant frequencies: a fixed resonant frequency (fᵣ) and a resonant frequency (fₘ) that varies with a load.

To achieve high-frequency and high-efficiency operation of an LLC resonant converter, it is necessary to create ZVS turn-on conditions for the MOSFETs (Q₁ and Q₂). For this purpose, the MOSFETs are turned on when their drain-source voltage (v_DS) is zero. Generally, the LLC resonant converter is designed with fᵣ > f > fₘ so that the MOSFETs (Q₁ and Q₂) in the square-wave generator turn on with ZVS.
For ZVS turn-off, either the MOSFET’s parasitic drain-source capacitance ($C_{oss}$) or an external small-value capacitor is used to reduce the $V_{DS}$ rise ramp of the MOSFET during turn-off and thereby turn on the MOSFET while $V_{DS}$ is low.
Operating mode of the LLC resonant converter

The LLC resonant converter controls its output voltage using frequency modulation. An equivalent circuit like the one shown in Figure 3.16 is used to calculate its input characteristics. (The equivalent load resistance and the actual load resistance differ.) Although an LLC resonant converter has a simple topology, its operation is complicated. Therefore, the following equivalent circuit is used to approximate the characteristics of the resonant circuit.

Figure 3.16 LLC resonant converter equivalent circuit
Calculating equivalent load resistance

The circuit on the primary side of an LLC resonant converter can be replaced with a sinusoidal power source ($i_{ac}$) as shown in Figure 3.16. An AC voltage with a square waveform ($v_{in2}$) appears at the input of the rectifier circuit on the secondary side. Because the average of $|i_{ac}|$ is equal to the output current ($i_o$), $i_o$ and $i_{ac}$ have the following relationship:

$$i_{ac} = \left(\frac{n \cdot i_o}{2}\right) \sin(\omega t) \quad \text{(square wave to sine wave)}$$

$v_{in2}$ is expressed as follows:

$$v_{in2} = +v_o \quad \text{(when } \sin(\omega t) > 0)$$

$$v_{in2} = -v_o \quad \text{(when } \sin(\omega t) < 0)$$

(where $v_o$ is the output voltage.)

The fundamental (sine-wave) component of $v_{in2}$ ($v_{in2ac}$) is expressed as follows: (The harmonic content of $v_{in2}$ has no effect on power transfer.)

$$v_{in2ac} = (4v_o/n) \sin(\omega t)$$

The AC equivalent load resistance ($R'_o$) observed on the primary side can be calculated as follows from the division of $v_{in2ac}$ by $i_{ac}$ and the turns ratios of the transformer ($n=N_1/N_2,3$)*1.

$$R'_o = n^2 \times \frac{v_{in2ac}}{i_{ac}}$$

$$R'_o = \left(\frac{8n^2}{\pi^2}\right) \cdot R_o$$

*1: The transformer winding voltage ratio is directly proportional to the winding turns ratio whereas the winding current ratio is inversely proportional to the winding turns ratio.

Resistance is voltage divided by current and proportional to the square of the winding turns ratio.

Input/output voltage ratio

In the AC equivalent circuit of the LLC resonant converter shown in Figure 3.16, $R'_o$ represents the equivalent load resistance described above.

($v_{in}$ is a sinusoidal approximation of $V_{in}$. $v'_o$ is obtained by converting $v_{in2ac}$ approximating sine wave of $V_o$ to the primary side.)

The relationship between $v_{in}'$ and $v'_o$ in Figure 3.16 can be calculated with a complex-valued function as follows:

$$Z_s = \frac{1}{j\omega C_r} + j\omega L_r, \quad Z_p = \frac{1}{j\omega L_m + \frac{1}{R_o}}$$

$$\frac{v'_o}{v_{in}} = \frac{Z_p}{Z_s + Z_p} = \frac{1}{1 + \frac{Z_s}{Z_p}} = \frac{1}{1 + \left(\frac{1}{j\omega C_r} + j\omega L_r\right)\left(\frac{1}{j\omega L_m + \frac{1}{R_o}}\right)} = \frac{1}{1 + \frac{L_r}{L_m} - \frac{1}{\omega^2 L_m C_r} + j\left(\frac{\omega L_r}{R_o} - \frac{1}{\omega C_r R_o}\right)}$$

Because
The above equation can be restated as:

\[ \frac{v_o'}{v_{in}} = \frac{1}{1 + \frac{L_r}{L_m} \left(1 - \frac{\omega_0^2}{\omega^2}\right) + jQ \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \]

Hence,

\[ \frac{|v_o'|}{|v_{in}|} = \frac{1}{\sqrt{\left(1 + \frac{L_r}{L_m} \left(1 - \frac{\omega_0^2}{\omega^2}\right)\right)^2 + Q^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}} \]

An LLC resonant converter has different operating modes above and below the maximum value of the input/output voltage ratio. The LLC resonant converter exhibits a capacitive operation in the region below the frequency at which it provides the maximum input/output voltage ratio. In this frequency region, the upper- and lower-arm MOSFETs are short-circuited. Generally, a capacitive operation is prevented by operating the LLC resonant converter at a higher frequency. The LLC resonant converter is not also used in this frequency region because of poor controllability since the output voltage changes only slightly in response to changes in f in the region where the switching frequency (f) is higher than the resonant frequency (f_r). Figure 3.17 shows the relationships between the input/output voltage ratio and the switching frequency.

1. When f=f_r, C_r and L_r are in series resonance. In this state, L_r has zero impedance, causing V_o' to be equal to V_{in}'. Therefore, the output voltage does not change in response to changes in the load resistance (R_0).
2. When f_m<f<f_r, the output voltage decreases under heavy load (with lower R_0').
3. When f approaches f_m, the output voltage increases. This is because, the L_m voltage, which increases as a result of series resonance between C_r and L_m+L_r, is applied to the transformer.
Primary-side current of the LLC resonant converter

The current on the primary side of the LLC circuit consists of a resonance current flowing through $L_r$ and $C_r$ and a resonance current flowing through $L_r+L_m$ and $C_r$.

![Waveforms of the primary-side current of the LLC resonant converter](image)

**Figure 3.18 Waveforms of the primary-side current of the LLC resonant converter**

Frequency and load of the LLC resonant converter

- **When the MOSFET switching frequency ($f$) is equal to the resonant frequency ($f_r$)**

  Figure 3.19 is a simplified version of the equivalent circuit shown in Figure 3.1. In this circuit, the input can be considered to be short-circuited with the series resonant circuit (consisting of $L_r$ and $C_r$) because they have the same frequency. The output voltage ($v_o'$) across the load resistor ($R_o'$) is approximately equal to $v_{in}'$. Therefore, the input/output voltage gain ($\eta$) is $v_o'/v_{in}' \approx 1$ regardless of the relationship of the load $R_o'$ and $f_m$ (i.e., the resonant frequency of $L_r+L_m$ and $C_r$).

![LLC resonant equivalent circuit](image)

**Figure 3.19 LLC resonant equivalent circuit**
• **When the load \(R_0'\) is heavy (i.e., when \(R_0'\) is small and the load current \(i_0\) is large)**

When \(\omega L_m >> R_0'\), the parallel circuit of \(L_m\) and \(R_0'\) can be considered to consist of only resistor \(R_0'\) as shown in Figure 3.20. Therefore, the resonant frequency \(f_m\) is close to \(f_r\) that is determined by the series resonance between \(L_r\) and \(C_r\). In this case, the input/output voltage gain \(\eta = v_o'/v_{in}'\) remains less than 1.

![Figure 3.20 Resonant circuit under heavy load (\(R_0'\))](image)

• **When the load \(R_0'\) is light (i.e., when \(R_0'\) is large and the load current \(i_0\) is small)**

When \(\omega L_m << R_0'\), the parallel circuit of \(L_m\) and \(R_0'\) can be considered to consist of only inductor \(L_m\) as shown in Figure 3.21. Therefore, \((L_r+L_m)\) and \(C_r\) form a series resonance circuit. The resonant frequency \(f_m\) deviates toward a lower value from \(f_r\) of the series resonance circuit consisting of \(L_r\) and \(C_r\). When \(R_0'\) is open (i.e., \(R_0' = \infty\)), \(f_m = \frac{1}{2\pi \sqrt{(L_r + L_m)C_r}}\). Therefore, the input/output voltage gain \(\eta = v_o'/v_{in}'\) becomes the maximum.

![Figure 3.21 Resonant circuit under light load (\(R_0'\))](image)

A resonant circuit generates an almost sinusoidal current from a square-wave voltage and feeds it to a rectifier circuit on the secondary side.

The output voltage \((v_o')\) can be changed by changing the switching frequency of the square wave generator.
4. Resonant inverters (induction heating and discharge tube drive)

This section describes induction heating and discharge tube drive, which are major applications of resonant inverters.

4.1. Induction heating

Induction heating (IH) is the process of heating an electrically conducting object by the application of an alternating magnetic field, which generates eddy currents that produce Joule heating. To generate a magnetic field, it is necessary to pass an AC current through a coil. An LC resonant circuit is used to apply an AC current to a coil. There are two resonance methods: voltage resonance (parallel LC circuit) and current resonance (series LC circuit).

**Voltage resonance**

Figure 4.1 shows a basic circuit and waveforms for voltage resonance. The following paragraphs describe its operation.

![Figure 4.1 Basic circuit and waveforms for voltage resonance](image)

#1. The IGBT is on. Current is flowing to the IGBT via the heating coil (L) from the filter capacitor (C₀) as well as through AC input—rectifier circuit—filter inductor (L₀). At this time, energy is stored in the heating coil (L).

#2. When the IGBT turns off, the IGBT current flowing through the heating coil (L) is cut off, causing current to flow from the heating coil (L) to the resonant capacitor (C) owing to the energy stored in L. The resonant capacitor is charged as the energy is released from the heating coil (L) and the resonance current gradually decreases. Consequently, the IGBT collector voltage increases sinusoidally.

#3. During period #3, the IGBT collector voltage decreases from the peak to 0 V. The resonance current of the heating coil reverses its direction from positive to negative. During this period, the resonant capacitor is discharged, causing the IGBT collector voltage to decrease.

#4. During period #4, the diode conducts. The diode conducts when the resonant capacitor (C) is discharged to a voltage equal to the C₀ voltage (with the IGBT collector voltage being less than 0 V). The LC resonant circuit resonates around the filter capacitor (C₀) voltage. When the IGBT...
collector voltage is less than 0 V, the voltage across the resonant capacitor (C) exceeds the voltage across the filter capacitor (C₀) in the reverse direction, causing the diode current to flow to the filter capacitor (C₀) via the heating coil (L). The IGBT is turned on during this period. The coil current becomes zero when all the energy stored in the heating coil (L) is released.

Steps #1 to #4 are repeated to pass a high-frequency AC current to the heating coil (L).
Current resonance

Figure 4.2 shows a basic circuit and waveforms for current resonance. The following paragraphs describe its operation.

#1. Q_H is off, and Q_L is on. Voltage is applied to the collector of Q_H whereas the collector voltage Q_L is almost zero. In this state, current flows through two paths: C_0—resonant capacitor (C_{RH})—heating coil (L)—Q_L and resonant capacitor (C_{RL})—heating coil (L)—Q_L.

#2. Q_L turns off. This causes the energy stored in the heating coil (L) to be released through the snubber capacitor (C_{SH}) and the resonant capacitor (C_{RH}). At the same time, C_{SL} is charged. As a result, during period #2, the Q_L voltage gradually increases with low switching loss. When the Q_L collector voltage exceeds the C_0 voltage (by V_F), the resonant inverter transitions to #3.

#3. As a result, D_H conducts, causing current to flow through two paths: D_H—C_{RH}—heating coil (L) and D_H—C_0—C_{RL}—heating coil (L). Q_H turns on while current is flowing through D_H.

#4. Current ceases to flow through D_H; instead, current flows through Q_H. Since Q_H is already on, it does not cause any switching loss. The resonant capacitor C_{RL} is charged through C_0—Q_H—heating coil (L) whereas the resonant capacitor C_{RH} is discharged through Q_H—heating coil (L). As a result, energy is stored in the heating coil (L).

#5. Q_H turns off. This causes the energy stored in the heating coil (L) to be released through the resonant capacitor C_{RL} and the snubber capacitor C_{SL}. At the same time, C_{SH} is charged. As a result, during period #5, the Q_H voltage gradually increases with low switching loss. When the Q_L collector voltage exceeds the C_0 voltage (by -V_F), the resonant inverter transitions to #6.

#6. As a result, D_L conducts, causing current to flow through two paths: D_L—heating coil (L)—C_{RL} and D_L—heating coil (L)—C_{RH}—C_0. Q_L turns on while current is flowing through D_L.

Steps #1 to #6 are repeated to pass a high-frequency AC current to the heating coil (L).
4.2. Discharge tube drive

Resonant circuits are also used in a lighting circuit for discharge tubes. Figure 4.3 shows an example of an inverter for hot-cathode fluorescent lamps (HCFLs) using an LC resonant circuit. As a side note, there are two types of fluorescent lamps: hot-cathode fluorescent lamps (HCFLs) and cold-cathode fluorescent lamps (CCFLs). HCFLs are used for general lighting whereas CCFLs are used for backlighting. To date, many fluorescent lamps have been replaced by LED lamps.

![Figure 4.3 Typical fluorescent lamp driver](image)

In Figure 4.3, a series resonance circuit consisting of a coil (L) and a capacitor (C₂) is formed after power-on. When the voltage across the resonant capacitor (C₂) is applied to the load (i.e., fluorescent lamp), it lights up.
Supplement 1: Leakage inductance and excitation inductance

Figure 4.4 shows a simplified structure of a transformer. $\Phi_m$ represents the main magnetic flux whereas $\Phi_{r1}$ and $\Phi_{r2}$ represent the leakage flux of the primary and secondary coils, respectively. Figure 4.5. shows an equivalent circuit of the transformer. $L_m$ is the excitation inductance caused by $\Phi_m$. $L_{r1}$ and $L_{r2}$ are the leakage inductances on the primary and secondary sides, which are caused by $\Phi_{r1}$ and $\Phi_{r2}$, respectively. $R_1$ and $R_2$ are the resistance of the primary and secondary coils, respectively, whereas $R_m$ is iron loss resistance. Figure 4.6 is an equivalent circuit obtained as a result of converting the elements on the secondary side to those on the primary side. The equations used for such conversion are given below:

$$L'_{r2} = L_{r2} \left(\frac{n_1}{n_2}\right)^2$$
$$R'_2 = R_2 \left(\frac{n_1}{n_2}\right)^2$$
$$R'_L = R_L \left(\frac{n_1}{n_2}\right)^2$$

where, $n_1$ is the number of turns of the primary coil, and $n_2$ is the number of turns of the secondary coil.
Supplement 2: Inductance measurement

- \((L_m + L_{r1})\) is measured with an LCR meter, with the outputs open-circuited as shown in Figure 4.7 to find excitation inductance \((L_m)\). Normally, \(L_{r1} << L_m\). Hence, \(L_m\) can be obtained by ignoring \(L_{r1}\).

- Leakage inductance \((L_r)\) is measured with the outputs short-circuited as shown in Figure 4.8. An LCR meter provides the value of \(L_{r1} + (1/(1/L_m + 1/L_{r2}'))\). Normally, \(L_{r2}' << L_m\). Hence, \(L_{r1} + L_{r2}'\) can be obtained by ignoring \(1/L_m\).

![Figure 4.7 Measurement of excitation inductance \((L_m)\)](image1)

![Figure 4.8 Measurement of leakage inductance \((L_r)\)](image2)
Supplemental 3: Coupling coefficient

The coupling coefficient of a transformer indicates what percentages of the self-inductances of its primary and secondary coils work as a transformer (i.e., excitation inductance) and what percentages work as a choke coil (i.e., leakage inductance).

The magnetic flux of the ideal transformer has no leakage flux and consists of only main flux. The ideal transformer has a coupling coefficient (k) of 1 (or -1). In reality, however, all transformers have leakage flux and therefore have a coupling coefficient of less than 1. Leakage flux creates inductances in series with the primary and secondary sides. These inductances are called leakage inductances, which act equivalently to the choke coils connected in series with the primary and secondary windings of a transformer.

Let the primary and secondary self-inductances be \( L_1 \) and \( L_2 \), respectively, and the primary and secondary effective inductances (excitation inductances) be \( L_{m1} \) and \( L_{m2} \), respectively. Then,

\[
L_{m1} = k \times L_1 \quad L_{m2} = k \times L_2
\]

Leakage inductances (\( L_{r1} \) and \( L_{r2} \)) are calculated as follows:

\[
L_{r1} = (1-k) \times L_1 \quad L_{r2} = (1-k) \times L_2
\]
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