Bipolar Transistors
Electrical and Equivalent Circuit

Description

This document describes the electrical characteristics of bipolar transistors.

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1. Transistor characteristics

Equivalent parameters of a transistor include the device parameters closely related to its internal operation and the circuit parameters that are represented as a matrix by treating the transistor as a four-terminal network.

Equivalent circuits are also divided into small-signal and large-signal equivalent circuits, depending on the amplitude of signals to be handled. Since there are numerous equivalent circuits, circuit designers should carefully consider the scopes and limitations of their applications. Table 1.1 categorizes equivalent circuits. Chapter 1 focuses on commonly used small-signal equivalent circuits.

Table 1.1 List of transistor equivalent circuits

Transistor equivalent circuits	Small-signal equivalent circuits (General linear circuits such as amplifiers, oscillators, modulators, and demodulators)	Device parameters	Early's T-type equivalent circuit (Common-base circuit) Giacoletto’s п-type equivalent circuit (Common-collector and common-emitter circuits)
		Circuit parameters	Matrices showing the relationship between the input and output by voltage and current a-b matrixes g-h matrices (low frequency) y-z matrices (high frequency) Matrices showing the relationship between the input and output by power s matrices (ultra-high frequency) (transmittance coefficient and reflection coefficient indications)
	Large-signal equivalent circuits - device parameters (Nonlinear circuits such as pulse, digital, and switching circuits)		Ebers-Moll current control model Beaufoy-Sparkes charge control model Linvil density control model Other nonlinear models

1.1. Device parameters

(1) Early's T-type equivalent circuit

Figure 1.1 shows Early's T-type equivalent circuit.

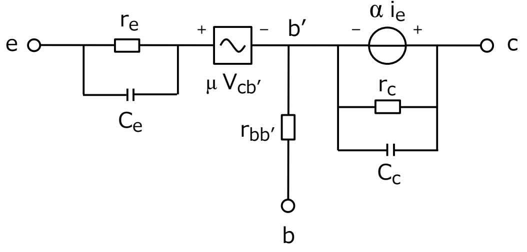


Figure 1.1 Early's T-type equivalent circuit

(a) r_e: Emitter resistance

r_e is the forward-bias resistance across the base-emitter junction, which is calculated as:

$$r_e = \frac{k\,T}{q\,I_E}\ (\Omega) \quad \cdots\cdots\cdots \quad (1\text{–}1)$$

k : Boltzmann constant (1.38×10^{-23} J/ K)

T : Absolute temperature (K)

q : Elementary charge (1.602×10^{-19} C)

I_E : Emitter current (A)

At room temperature (300 K), Equation 1-1 is restated as follows when the emitter current is given in mA:

$$r_e \approx \frac{26}{I_E\ (\mathrm{mA})}\ (\Omega) \quad \cdots\cdots\cdots \quad (1\text{–}2)$$

(b) C_e: Emitter capacitance ($C_{Te}+C_{De}$)

The emitter capacitance is the sum of the depletion capacitance C_{Te} and the diffusion capacitance C_{De} in the base-emitter junction. The depletion layer capacitance in the base-emitter junction can be ignored since it is far smaller than the diffusion capacitance. The depletion layer capacitance C_{Te} and the diffusion capacitance C_{De} can be calculated using Equation 1-3 and Equation 1-4 respectively:

$$C_{Te} = A_e \sqrt[3]{\frac{\frac{1}{2}\,\varepsilon\, q\, {}^{n}N}{\phi_0 - V_{b'e}}} \ (\mathrm{F}) \qquad \cdots\cdots \quad (1\text{–}3)$$

A_e : Emitter junction area (m^2)

ε : Dielectric constant

${}^{n}N$: Majority carrier density (m^{-3}) on the side with higher specific resistance (NPN in this case)

Φ_0 : Contact potential difference (potential barrier in thermodynamic equilibrium) (V)

$V_{b'e}$: Voltage applied across the base-emitter junction (V)

$$C_{De} = \frac{q\, I_E\, W^2}{2\, k\, T\, D} \ (\mathrm{F}) \qquad \cdots\cdots \quad (1\text{–}4)$$

W : Base width (m)

D : Diffusion coefficient of minority carriers in the base layer (m^2/ s)

(c) μ : Voltage feedback ratio (Early constant)

This constant due to the Early effect is a base-width modulation parameter.

$$\mu = \frac{k\, T\, d_C}{3\, q\, W \left(\phi_0 - V_{b'e} \right)} \ (\mathrm{F}) \qquad \cdots\cdots \quad (1\text{–}5)$$

d_C : Width of the collector depletion layer (m)

(d) r_c : Collector resistance

This is a base-width modulation parameter, which is represented as:

$$r_C = \frac{1}{I_E \left(\dfrac{\partial \alpha}{\partial V_{b'c}} \right)} \ (\Omega) \qquad \cdots\cdots \quad (1\text{–}6)$$

r_c is typically 1 to 2 MΩ.

(e) Cc : Collector capacitance

As is the case with the emitter capacitance, the collector capacitance is the sum of the depletion layer capacitance C_{TC} and the diffusion capacitance C_{DC} in the collector-base junction.

The diffusion capacitance in the collector-base junction can be ignored since it is far smaller than the depletion layer capacitance. The depletion layer capacitance can be calculated as:

$$C_{TC} = A_C \sqrt[3]{\frac{\frac{\varepsilon^2 \ q \ a}{12}}{\phi_0 - V_{b'c}}} \ (F) \qquad \cdots\cdots (1\text{–}7)$$

A_C : Collector junction area (m^3)

a : Dopant concentration gradient (m^{-4})

$V_{b'c}$: Voltage applied across the base-collector junction (V)

C_{TC} is typically 1 to 10 pF.

(f) α : DC current gain

This is the only parameter of Early's T-type equivalent circuit that exhibits frequency dependence and can be calculated as:

$$\alpha = \frac{\alpha_0}{1 + j\,\omega\, C_e\, r_c}$$

$$f_\alpha = \frac{1}{2\,\pi\, C_e\, r_e}$$

Hence:

$$\alpha = \frac{\alpha_0}{1 + j\,\frac{f}{f_\alpha}} \qquad \cdots\cdots (1\text{–}8)$$

α_0 : Value of α at low frequency

f_α : α cut-off frequency (frequency at which α drops by 3 dB)

Figure 1.2 shows the frequency locus of α. The measurement of α reveals that the difference between theoretical and measured values increases as the frequency approaches f_α. This is because Early's T-type equivalent circuit is based on primary approximation of physical phenomena.

T$_o$ correct this error, Thomas-Moll included the excess phase parameter m in the equation:

$$\alpha = \frac{\alpha_0}{1 + j\frac{f}{f_\alpha}} e^{-jm\frac{f}{f_\alpha}} \quad \text{(1–9)}$$

This equation matches well with measured values at frequencies lower than f_α.

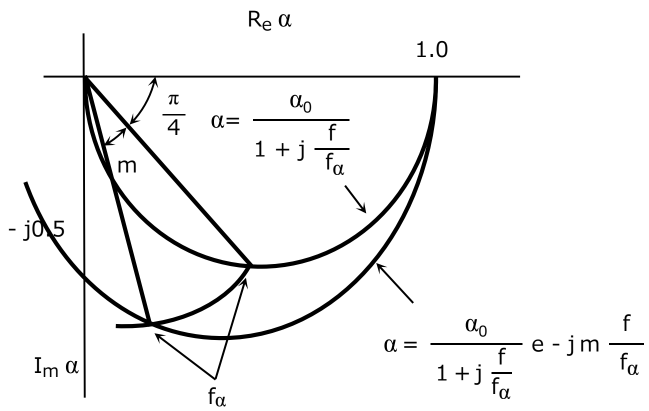
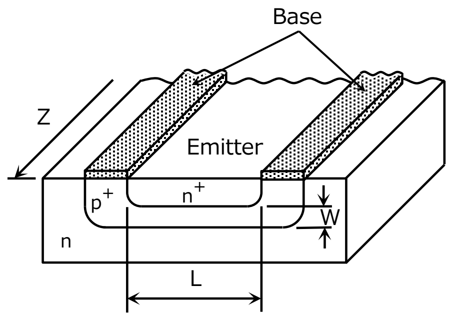


Figure 1.2 Frequency locus of α

(g) $r_{bb'}$: Base spreading resistance

This is the resistance from the center of the base layer to the external base terminal that contributes to the operation of a transistor and is determined by the shape and dimensions of the transistor and the specific resistance of the base layer. The comb-shaped base spreading resistance can be calculated as follows.



$$r_{bb'} \approx \frac{1}{12} \frac{\rho_B}{W} \frac{L}{Z} \ (\Omega) \quad \text{(1–10)}$$

ρ_B Specific resistance of the base layer ($\Omega \cdot m$)

In a common-emitter configuration, the DC current gain (β) of a transistor is represented as follows using π-type equivalent circuit:

$$\beta = \frac{\alpha_0}{1 - \alpha_0}\left(\frac{1}{1 + j\,\omega\,C_{b'e}\;r_{b'e}}\right) = \frac{\beta_0}{1 + j\,\omega\,C_{b'e}\;r_{b'e}}$$

As is the case with f_α, let's define the β cut-off frequency f_β as the frequency at which the absolute value of β equals $\beta_0/\sqrt{2}$. Then, f_β is calculated as:

$$f_\beta = \frac{1}{2\,\pi\,C_{b'e}\;r_{b'e}}$$

$$\beta = \frac{1}{1 + j\,\frac{f}{f_\beta}} \quad \cdots\cdots (1\text{–}11)$$

(2) π-type equivalent circuit

Figure 1.3 shows the π-type equivalent circuit, which is essentially the same as the T-type equivalent circuit described above. The π-type equivalent circuit differs from the T-type equivalent circuit only in that, in principle, the parameters of the former have no frequency response.

Since the physical meaning of each parameter is easy to understand, the π-type equivalent circuit is widely used. To use it for circuit calculation, it is convenient to simplify the basic configuration shown in Figure 1.3, considering the frequency range.

Table 1.2 shows the relationships of the parameters of the T-type and the π-type equivalent circuits.

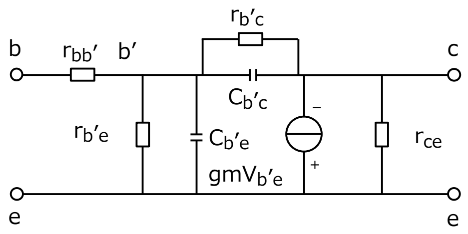


Figure 1.3 π-type equivalent circuit

Table 1.2 Relationships between the parameters of the T-type and the π-type equivalent circuits

T-type equivalent circuit	π-type equivalent circuit
C_e	$C_{b'e}$
$\dfrac{r_e}{1 - \alpha_0}$	$r_{b'e}$
C_c	$C_{b'c}$
$\dfrac{1}{r_e} - \dfrac{\mu (1 - \alpha_0)}{r_e}$	$\dfrac{1}{r_{b'c}}$
$\dfrac{r_e}{\mu}$	r_{ce}
$\dfrac{\alpha_0}{r_e}$	g_m
$r_{bb'}$	$r_{bb'}$

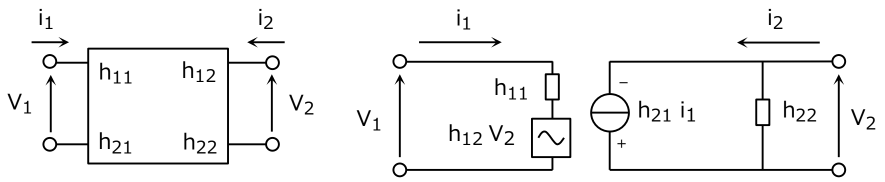
1.2. Circuit parameters

(1) Matrices showing the relationships between the input and the output by voltage and current

This method regards a transistor as a four-terminal circuit network to describe it only with the electrical characteristics of its terminals irrespective of the physical characteristics of the transistor.

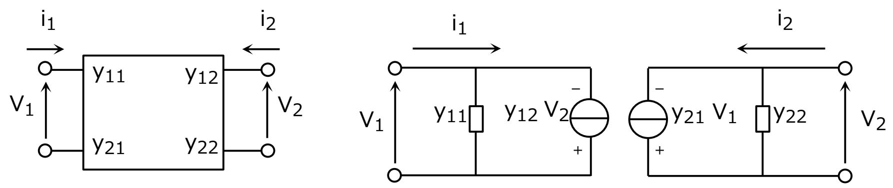
There are six types of matrices (a, b, g, h, y and z) that represent the relationships among the input and output voltages and currents. Of the six types, the h and y matrices are used relatively frequently.

Figure 1.4 and Figure 1.5 show the definitions of the h and y matrices. The suffixes e and b following the letters i, r, f, and o distinguish between the common-emitter and common-base configurations.



$$\begin{bmatrix} V_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \begin{bmatrix} i_1 \\ V_2 \end{bmatrix}$$

Figure 1.4 Circuit network using the h matrix



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_i & y_r \\ y_f & y_o \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Figure 1.5 Circuit network using the y matrix

The parameters in the matrices have the following meanings:

h_i :	Input impedance	y_i :	Input admittance
h_r :	Reverse voltage feedback ratio	y_r :	Reverse transfer admittance
h_f :	Forward current gain	y_f :	Forward transfer admittance
h_o :	Output admittance	y_o :	Output admittance

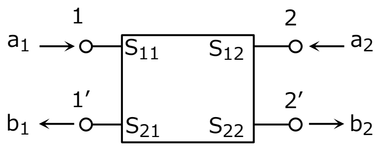
The h matrices are often used for low-frequency regions whereas the y matrices are commonly used for high-frequency regions.

(2) Matrix showing the relationships between the input and the output by power

The S matrices (scattering matrices) are commonly used to represent the phenomena in microwave circuits such as the reflection and transmission of waves.

As the frequency limits of semiconductor devices increase, the S matrices are sometimes used to describe their circuit parameters.

Figure 1.6 shows the definitions of the S matrix.



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} S_i & S_r \\ S_f & S_o \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Figure 1.6 Circuit network using the S matrix

Each parameter has the following meaning:

S_{11} : Input reflection coefficient

S_{12} : Reverse transmission coefficient

S_{21} : Forward transmission coefficient

S_{22} : Output reflection coefficient

As is the case with the h and y matrices, the suffixes e and b denote the common-emitter and common-base configurations respectively.

Table 1.3 Interrelation of parameters

	[h]	[y]	[s]
[h]	$\begin{matrix} h_i & h_r \\ h_f & h_o \end{matrix}$	$\begin{matrix} \frac{1}{y_i} & -\frac{y_r}{y_i} \\ \frac{y_f}{y_i} & \frac{y_i y_o - y_r y_f}{y_i} \end{matrix}$	$\begin{matrix} \frac{(1+s_i)(1+s_o)-s_r s_f}{(1-s_i)(1+s_o)+s_r s_f} & \frac{2s_r}{(1-s_i)(1+s_o)+s_r s_f} \\ \frac{-2s_f}{(1-s_i)(1+s_o)+s_r s_f} & \frac{(1-s_i)(1-s_o)-s_r s_f}{(1-s_i)(1+s_o)+s_r s_f} \end{matrix}$
[y]	$\begin{matrix} \frac{1}{h_i} & -\frac{h_r}{h_i} \\ \frac{h_f}{h_i} & \frac{h_i h_o - h_r h_f}{h_i} \end{matrix}$	$\begin{matrix} y_i & y_r \\ y_f & y_o \end{matrix}$	$\begin{matrix} \frac{(1-s_i)(1+s_o)+s_r s_f}{(1+s_i)(1+s_o)-s_r s_f} & \frac{-2s_r}{(1+s_i)(1+s_o)-s_r s_f} \\ \frac{-2s_f}{(1+s_i)(1+s_o)-s_r s_f} & \frac{(1+s_i)(1-s_o)+s_r s_f}{(1+s_i)(1+s_o)-s_r s_f} \end{matrix}$
[s]	$\begin{matrix} \frac{(h_i-1)(h_o+1)-h_r h_f}{(h_i+1)(h_o+1)-h_r h_f} & \frac{2h_r}{(h_i+1)(h_o+1)-h_r h_f} \\ \frac{-2h_f}{(h_i+1)(h_o+1)-h_r h_f} & \frac{(1+h_i)(1-h_o)+h_r h_f}{(h_i+1)(h_o+1)-h_r h_f} \end{matrix}$	$\begin{matrix} \frac{(1-y_i)(1+y_o)+y_r y_f}{(1+y_i)(1+y_o)-y_r y_f} & \frac{-2y_r}{(1+y_i)(1+y_o)-y_r y_f} \\ \frac{-2y_f}{(1+y_i)(1+y_o)-y_r y_f} & \frac{(1+y_i)(1-y_o)+y_r y_f}{(1+y_i)(1+y_o)-y_r y_f} \end{matrix}$	$\begin{matrix} s_i & s_r \\ s_f & s_o \end{matrix}$

Table 1.4 Conversion formulas for h parameters

Known h parameters		Converted h parameters: Common-base		Common-emitter		Common-collector	
Common-base				$\frac{h_{ib}}{1 + h_{fb}}$	$\frac{\Delta h_b - h_{rb}}{1 + h_{fb}}$	$\frac{h_{ib}}{1 + h_{fb}}$	1
				$\frac{-h_{fb}}{1 + h_{fb}}$	$\frac{h_{ob}}{1 + h_{fb}}$	$\frac{-1}{1 + h_{fb}}$	$\frac{h_{ob}}{1 + h_{fb}}$
Common-emitter		$\frac{h_{ie}}{1 + h_{fe}}$	$\frac{\Delta h_e - h_{re}}{1 + h_{fe}}$			h_{ie}	$1 - h_{re}$
		$\frac{-h_{fe}}{1 + h_{fe}}$	$\frac{h_{oe}}{1 + h_{fe}}$			$-(1 + h_{fe})$	h_{oe}
Common-collector		$\frac{-h_{ic}}{h_{fc}}$	$\frac{-\Delta h_{rc}}{h_{fc}} - 1$	h_{ic}	$1 - h_{rc}$		
		$\frac{-(1 + h_{fc})}{h_{fc}}$	$\frac{-h_{oc}}{h_{fc}}$	$-(1 + h_{fc})$	h_{oc}		

$\Delta h_e = h_{ie} h_{oe} - h_{re} h_{fe}$ 、 $\Delta h_b = h_{ib} h_{ob} - h_{rb} h_{fb}$ 、 $\Delta h_c = h_{ic} h_{oc} - h_{rc} h_{fc}$

Table 1.5 Conversion formulas for y parameters

		Converted y parameters		
		Common-base	Common-emitter	Common-collector
Known y parameters	Common-base		$\begin{matrix} \sum y_b & -(y_{rb} + y_{ob}) \\ -(y_{fb} + y_{ob}) & y_{ob} \end{matrix}$	$\begin{matrix} \sum y_b & -(y_{ib} + y_{fb}) \\ -(y_{ib} + y_{rb}) & y_{ib} \end{matrix}$
	Common-emitter	$\begin{matrix} \sum y_e & -(y_{re} + y_{oe}) \\ -(y_{fe} + y_{oe}) & y_{oe} \end{matrix}$		$\begin{matrix} y_{ie} & -(y_{ie} + y_{re}) \\ -(y_{ie} + y_{fe}) & \sum y_e \end{matrix}$
	Common-collector	$\begin{matrix} y_{oc} & -(y_{fc} + y_{oc}) \\ -(y_{rc} + y_{oc}) & \sum y_c \end{matrix}$	$\begin{matrix} y_{ic} & -(y_{ic} + y_{rc}) \\ -(y_{ic} + y_{fc}) & \sum y_c \end{matrix}$	

$\Sigma y_e = y_{ie} + y_{re} + y_{fe} + y_{oe}$

$\Sigma y_b = y_{ib} + y_{rb} + y_{fb} + y_{ob}$

$\Sigma y_c = y_{ic} + y_{rc} + y_{fc} + y_{oc}$

Table 1.6 h parameters converted using T-type equivalent circuit

Common-base		Common-emitter	
h_{ib}	$\dfrac{r_e + r_{bb'}\left[(1-\alpha_0) + j\dfrac{f}{f_\alpha}\right]}{1 + j\dfrac{f}{f_\alpha}}$	h_{ie}	$r_{bb'} + \dfrac{r_e}{(1-\alpha_0) + j\dfrac{f}{f_\alpha}}$
h_{rb}	$j\,2\pi f\,C_c\,r_{bb'}$	h_{re}	$2\pi f_\alpha C_c r_e \dfrac{j\dfrac{f}{f_\alpha}}{(1-\alpha_0) + j\dfrac{f}{f_\alpha}}$
h_{fb}	$\dfrac{-\alpha_0}{1 + j\dfrac{f}{f_\alpha}}$	h_{fe}	$\dfrac{\alpha_0}{(1-\alpha_0) + j\dfrac{f}{f_\alpha}}$
h_{ob}	$j\,2\pi f\,C_c$	h_{oe}	$2\pi f_\alpha C_c \dfrac{j\dfrac{f}{f_\alpha}\left(1 + j\dfrac{f}{f_\alpha}\right)}{(1-\alpha_0) + j\dfrac{f}{f_\alpha}}$

Table 1.7 y parameters converted using T-type equivalent circuit

Common-base		Common-emitter	
y_{ib}	$\dfrac{1 + j\dfrac{f}{f_\alpha}}{r_e + j\,r_{bb'}\dfrac{f}{f_\alpha}}$	y_{ie}	$\dfrac{(1-\alpha_0) + j\dfrac{f}{f_\alpha}}{r_e + j\,r_{bb'}\dfrac{f}{f_\alpha}}$
y_{rb}	$-2\pi f_\alpha C_c \dfrac{j\dfrac{f}{f_\alpha}\left(1 + j\dfrac{f}{f_\alpha}\right)}{\dfrac{r_e}{r_{bb'}} + j\dfrac{f}{f_\alpha}}$	y_{re}	$-2\pi f_\alpha C_c \dfrac{r_e}{r_{bb'}} \dfrac{j\dfrac{f}{f_\alpha}}{\dfrac{r_e}{r_{bb'}} + j\dfrac{f}{f_\alpha}}$
y_{fb}	$-\dfrac{\alpha_0}{r_e + j\,r_{bb'}\dfrac{f}{f_\alpha}}$	y_{fe}	$\dfrac{\alpha_0}{r_e + j\,r_{bb'}\dfrac{f}{f_\alpha}}$
y_{ob}	$2\pi f_\alpha C_c \dfrac{j\dfrac{f}{f_\alpha}\left(1 + \dfrac{r_e}{r_{bb'}} + j\dfrac{f}{f_\alpha}\right)}{\dfrac{r_e}{r_{bb'}} + j\dfrac{f}{f_\alpha}}$	y_{oe}	Same as for y_{ob}

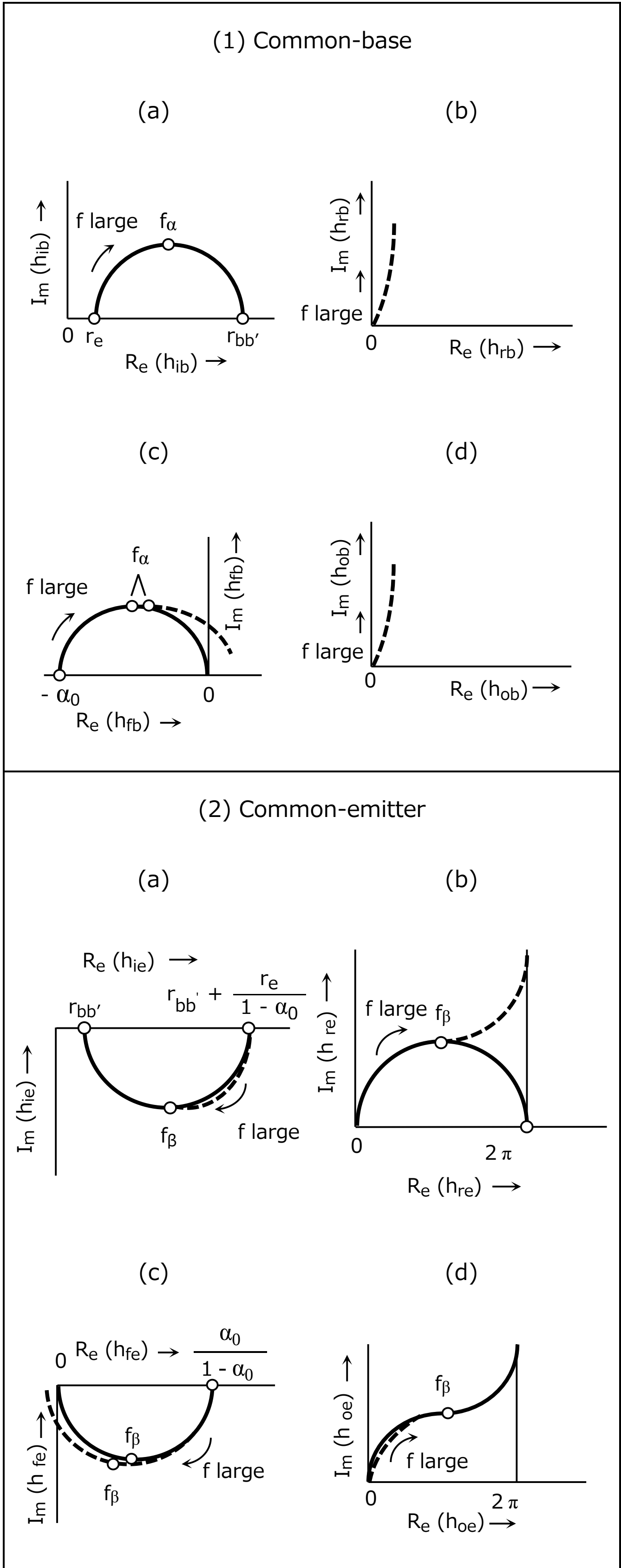


Figure 1.7 Frequency locus of h parameters

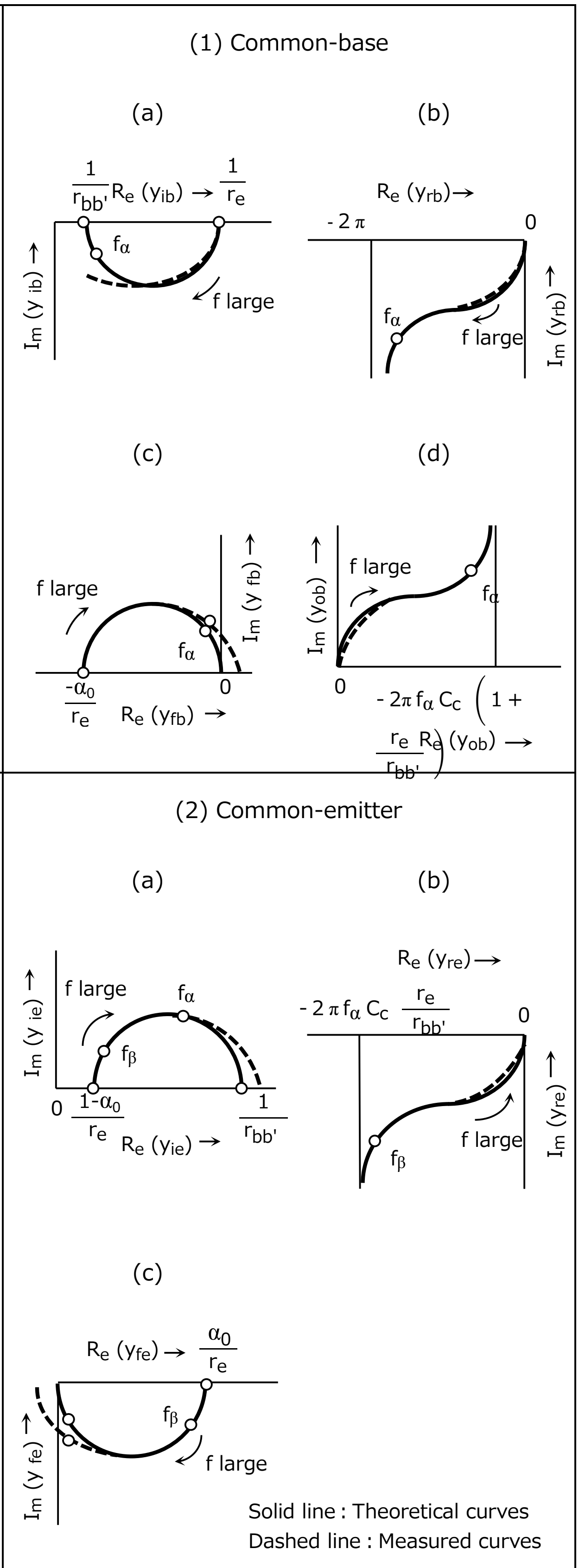


Figure 1.8 Frequency locus of y parameters

See Table 1.3 to Table 1.5 for the relationships among the circuit parameters and the conversion between the common-base and common-emitter parameters. Figure 1.7 and Figure 1.8 show the frequency loci of the h and y parameters obtained from Table 1.6 and Table 1.7 respectively. The parameters described above vary with the operating point and temperature. Circuit designers should understand their effects on the parameters.

1.3. Low-frequency, low-noise amplifiers

(1) Designing low-noise amplifiers

It is necessary to select and use transistors carefully when designing low-noise amplifiers.

Voltage, current, and signal source impedance should be considered to ensure that the transistors are used within the ranges that exhibit the best performance of the transistors. To help circuit designers obtain the best performance from low-noise transistors, this section describes the concept of noise characteristics, the optimal conditions of transistors, and the relationships between the noise figures of transistors and the S/N ratios of amplifiers.

(2) Noise characteristics of transistors

The noise figure (NF) of a transistor is given by:

$$NF = 10 \log \left(\frac{E_{si}}{E_{ni}} \Big/ \frac{E_{so}}{E_{no}} \right)^2$$

$$= 20 \log \left(\frac{E_{si}}{\sqrt{4\,k\,T\,R_g\,B}} \Big/ \frac{E_{so}}{E_{no}} \right) \quad \cdots\cdots \quad (1\text{–}12)$$

E_{si} : Input signal voltage
E_{ni} : Input noise voltage
E_{so} : Output signal voltage
Eno : Output noise voltage
k : Boltzmann constant (1.38×10^{-23} J/ °K)
T : Absolute temperature (K)
R_g : Signal source resistance
B : Noise bandwidth (Hz)
or $E_{ni} = \sqrt{4\,k\,T\,R_g\,B}$

Figure 1.9 shows the NF-vs-frequency curve, which is divided into three regions: 1) 1/ f region, 2) white noise region, and 3) f^2 noise region.

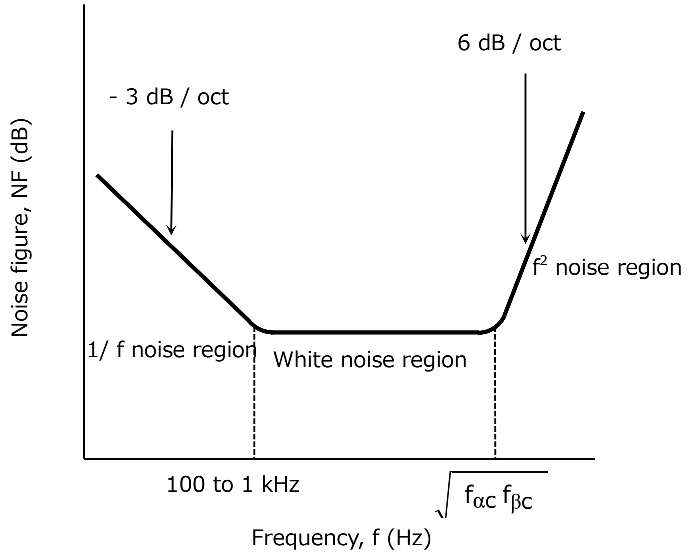
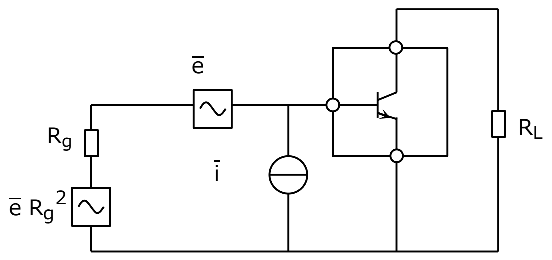


Figure 1.9 Relationship between NF and frequency

Table 1.8 Types of noise

Type / Item	1/ f noise region	White noise region	f^2 noise region
Description	Noise decreases at -3 dB/ oct in proportion to frequency f.	Noise remains constant over a range of frequency.	Noise increases at 6 dB/ oct in proportion to frequency f.
Cause	Surface fluctuation	Thermal noise caused by the base spreading resistance $r_{bb'}$	Fluctuation caused by current separation
Audio applications	Noise generated	Noise generated	Not noise generated

A transistor can be modeled with a voltage noise source (e_N) and a current noise source (i_N) as shown below.



$$e_N = \sqrt{4\,k\,T\,R_N\,B}$$

$$i_N = \sqrt{2\,q\,I_b\,B}$$

R_N : Equivalent noise resistance (Ω)

q : Elementary charge 1.602×10^{-19} (C)

Figure 1.10 Noise source of transistor

Considering the ideal transistor without any noise source, the noise figure (NF) is given by:

$$NF = 10 \log \left(\frac{4\,k\,T\,R_g + e_N^2 + i_N^2 + R_g^2 + 2\,\gamma\, e_N\, i_N}{4\,k\,T\,R_g} \right) \quad \cdots\cdots\cdots\cdots (1\text{–}13)$$

B : 1Hz

γ : Correlation function of e_N and i_N

Equation 1-13 shows that NF is a function of e_N and i_N.

It is evident from Equation 1-13 that the noise figure NF is dependent on the collector current I_C and the signal source impedance R_g. Let the total noise voltage be e_{NT}. Then,

$$\bar{e}_{NT}^2 = 4\,k\,T\,R_g + e_N^2 + i_N^2\,R_g^2 + 2\,\gamma\,e_N\,i_N \quad \cdots\cdots\cdots\cdots (1\text{–}14)$$

Figure 1.11 shows the relationship between the total noise voltage and the signal source impedance R_g.

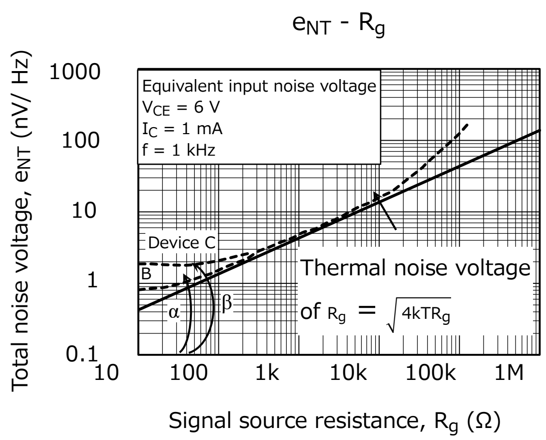


Figure 1.11 Total noise voltage – Signal source resistance

Referring to the curve of Device C in Figure 1.11, the noise figure can be seen as a difference (B) between its noise voltage and the thermal noise at R_g = 100 Ω.

$$NF = 20 (\log\beta - \log\alpha) \quad \rightarrow \quad \text{B in Figure 1.11}$$

As can be seen from Equation 1-14, voltage noise is more dominant in the small R_g region. However, current noise is dominant in the region where Rg increases.

R_g, I_C, and noise figure can be shown by plotting contours of the constant noise figure as shown in Figure 1.12 and Figure 1.13.

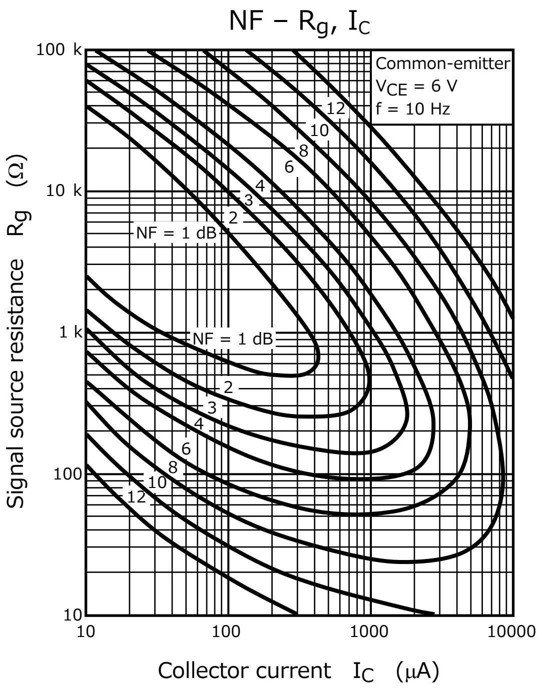


Figure 1.12 NF – R_g, I_C (1)

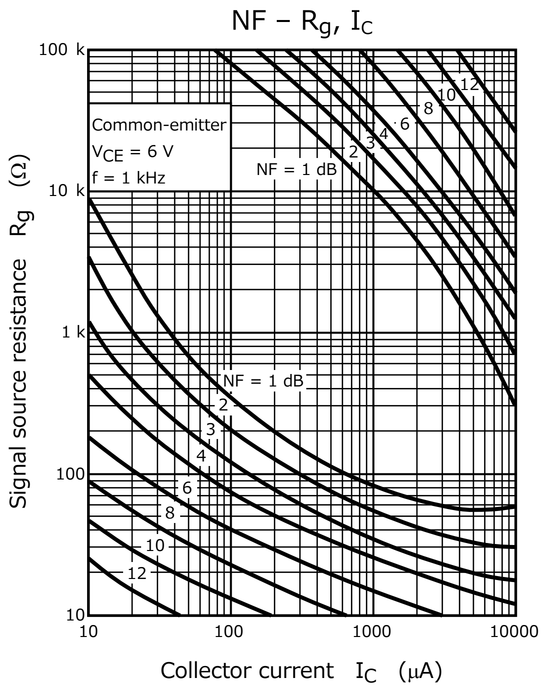


Figure 1.13 NF – R_g, I_C (2)

These noise figure contours can be used to determine the optimal usage condition of an amplifier.

Use the signal source impedance of the amplifier to obtain the collector current I_C at which the noise figure is minimum from the noise figure contours at f = 1 kHz and f = 10 Hz. When designing a low-noise amplifier, it is necessary to consider the conditions of the circuits preceding and following the amplifier. The next subsection describes an amplifier's noise, considering the foregoing.

(3) Amplifier noise

The signal-to-noise (SN) ratio is an important factor in designing an amplifier.

$$SN = 20 \log \frac{\text{Rate output}}{\text{Output noise voltage}} \quad (\text{dB}) \qquad \cdots\cdots (1\text{–}15)$$

From Equation 1-12, Equation 1-15 can be restated as follows to include NF.

$$SN = 20 \log \frac{E_{so}}{E_{no}}$$

$$= 10 \log \frac{E_{so}{}^2}{E_{no}{}^2}$$

$$= 10 \log \left(\frac{E_{si}{}^2}{E_{no}{}^2} 10^{\frac{NF}{10}} \right)$$

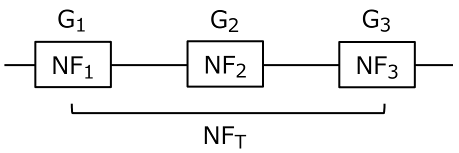
$$= 10 \log \frac{E_{si}{}^2}{4\,k\,T\,R_g\,B} - NF \quad (\text{dB}) \qquad \cdots\cdots (1\text{-}16)$$

$$\boxed{\text{Amplifier's S/ N ratio (dB)}} = \boxed{\text{Input S/ N ratio (dB)}} - \boxed{\text{Amplifier's NF (dB)}}$$

Noise figure of multi-stage amplifiers

The noise figure of a multi-stage amplifier like the one shown in Figure 1.14 can be calculated as follows:

$$NF_T = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1\,G_2} \qquad \cdots\cdots (1\text{–}17)$$

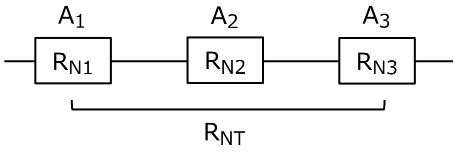


- NF_T : Total noise figure
- NF_1 : Noise figure of the first amplifier
- NF_2 : Noise figure of the second amplifier
- NF_3 : Noise figure of the third amplifier
- G1 : Power gain of the first amplifier
- G2 : Power gain of the second amplifier
- G3 : Power gain of the third amplifier

Figure 1.14 Noise figure of a multi-stage amplifier

The equivalent noise resistance (R_N) of this amplifier is:

$$R_N = R_{N1} + \frac{R_{N2}}{A_1} + \frac{R_{N3}}{(A_1 A_2)^2} \quad \cdots\cdots (1\text{–}18)$$



R_{NT} : Total equivalent input noise resistance
R_{N1} : Equivalent noise resistance of the first amplifier
R_{N2} : Equivalent noise resistance of the second amplifier
R_{N3} : Equivalent noise resistance of the third amplifier
A1 : Power gain of the first amplifier
A2 : Power gain of the second amplifier

Figure 1.15 Equivalent noise resistance of a multi-stage amplifier

Equation 1-17 and Equation 1-18 indicate that, if the power gain of the first amplifier (A_1) is sufficiently large, the total noise figure NF_T is:

$$NF_T \approx NF_1 \quad \cdots\cdots (1\text{–}19)$$

The total noise figure of the multi-stage amplifier is close to that of the first amplifier.

Calculating the total noise figure NF_T of a multi-stage amplifier from the nominal NF parameters of transistors

The NF values in the transistor datasheets are the measurements taken at spot frequencies (such as 1 kHz, 100 Hz, and 10 Hz). These values cannot be used without adjustment to design a wide-bandwidth amplifier with low-frequency boost. Since the f^2 noise region lies in the high-frequency region, only the 1/ f and white noise regions are related to low-frequency amplification.

Assuming:

$\bar{e}_g{}^2$: Mean square voltage of the thermal noise generated by signal source resistance R_g
$\bar{e}_w{}^2$: Mean square voltage of white noise
$\bar{e}^2_{1/f}$: Mean square voltage of 1/ f noise
$\cdots\cdots (1\text{–}20)$

the following equation is derived from the definition of the noise figure:

NF (white noise region) =

$$\frac{\bar{e}_g{}^2 + \bar{e}_w{}^2}{\bar{e}_g{}^2} = NF_{(1kHz)} \quad \cdots\cdots (1\text{–}21)$$

$NF_{(1kHz)}$: NF at the 1-kHz spot frequency

$\bar{e}_W^{\,2}$ is calculated as follows from Equation 1-20:

$$\bar{e}_W^{\,2} = (\,NF_{(1kHz)} - 1\,)\,\bar{e}_g^{\,2} \quad \cdots\cdots \quad (1\text{-}22)$$

Let the noise figure at 10 Hz be $NF_{(10Hz)}$. Then,

$$NF_{(10Hz)} = \frac{\bar{e}_g^{\,2} + \bar{e}_W^{\,2} + \bar{e}^{\,2}_{1/f\,(10\,Hz)}}{\bar{e}_g^{\,2}} \quad \cdots\cdots \quad (1\text{–}23)$$

From Equation (1-22),

$$\frac{\bar{e}^{\,2}_{1/f\,(10\,Hz)}}{\bar{e}_g^{\,2}} = (\,NF_{(10\,Hz)} - NF_{(1\,kHz)}\,) \quad \cdots\cdots \quad (1\text{–}24)$$

Since the 1/ f noise decreases at -3 dB/ oct in proportion to frequency, $\bar{e}^{\,2}_{1/f}$ at a normal frequency can be calculated as follows:

$$\bar{e}^{\,2}_{1/f} = (\,NF_{(10\,Hz)} - NF_{(1\,kHz)}\,)\ \bar{e}_g^{\,2}\ \frac{10}{f} \quad \cdots\cdots \quad (1\text{–}25)$$

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1) WILLIAM A.RHEINFELDER : DESIGN OF LOW NOISE TRANSISTOR INPUT CIRCUITS, LONDON ILIFFE BOOKS LTD. (1964)

2) J.WATSON : SEMICONDUCTOR CIRCUIT DESIGN, ADAM HILGE LTD. (1970)

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